
Simultaneous Estimation of Multinomial Logistic Regression Models: Factor Analysis of Polytomous Item Response Data

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- Flexible **latent variable models** for **multivariate, polytomous nominal and/or ordinal variables** with **collateral information**.

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- **Observed discrete (multicategory) variables**, e.g.,
 - ◆ Multiple choice test items.
 - ◆ Surveys items.
 - ◆ “Likert”.
 - ◆ Coded qualitative data.

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- **Collateral Information** includes observed covariates (discrete or continuous) that are measures of individuals, items, response options, latent variables, as well as specific hypotheses about items and response options.

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- **(Multiple) Unobserved continuous variables**.

American National Elections Pilot (1998)

A: Who has the final responsibility to decide if a law is constitutional or not?

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B: Whose responsibility is it to nominate judges to the Federal courts?

President (50.8%), Congress (15.6%), Supreme Court (25.5%), Don't know (8.2%)

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Republicans (70.3%), Democrats (16.0%), Don't know (13.7%)

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Republicans (70.3%), Democrats (16.0%), Don't know (13.7%)

D: Do you happen to know which party has the most members in the U.S. Senate?

Republicans (62.8%), Democrats (19.0%), Don't know (18.2%)

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Anderson, C.J., Verkuilen, J.V., & Peyton, B.L. (accepted).
<http://faculty.ed.uiuc.edu/cja/homepage> → “Publications”.

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- Ordering of response the options unknown.

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- Dealing with “Don’t Know”.

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- Different number of response options.
- Scoring of responses needed.

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- Latent variable structure unknown.

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 - ◆ One dominant underlying dimension of “Knowledge.”

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 - ◆ One dominant underlying dimension of “Knowledge.”
 - ◆ Two correlated latent variables: Structure of political system (items A & B) and Party in Power (items C & D)

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- Latent variable structure unknown.
 - ◆ One dominant underlying dimension of “Knowledge.”
 - ◆ Two correlated latent variables: Structure of political system (items A & B) and Party in Power (items C & D)
- How do the instructions given to respondents affect all of this?

Major Existing Approaches

to Latent Variable Modeling of Discrete Response Data:

- Quantify the data and then use factor analysis (or SEM) for continuous data.

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 - ◆ Need to know the order of the response options.
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- Factor analysis of discrete data (Bartholomew, Steele, Moustaki & Galbraith, 2008)

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- Quantify the data and then use factor analysis (or SEM) for continuous data.
 - ◆ Need to know the order of the response options.
 - ◆ Does not allow for alternative scoring for different latent variables.
- Item response theory (e.g., Bock's multinomial response model)
 - ◆ Multiple latent variables is a problem for standard estimation algorithms (i.e., numerical integration).
- Factor analysis of discrete data (Bartholomew, Steele, Moustaki & Galbraith, 2008)
 - ◆ Lack of available of software and flexibility of implementation.
 - ◆ Methods and programs for nominal data are sorely lacking and "...work on ordinal categorical variables is nearer the research frontier and is consequently more incomplete, and in some sense, more difficult than other methods." (p. 243)

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Conditional Specification Approach

or the **Item Response Theory Approach**

1. Bock's nominal response model, e.g.,

$$P(Y_i = j_i | \theta) = \frac{\exp[\lambda_{ij_i} + \nu_{ij_i} \theta]}{\sum_{h_i} \exp[\lambda_{ih_i} + \nu_{ih_i} \theta]}.$$

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2. If we knew or had estimates of the ν_{ij_i} 's for all the items, then $\sum_i \nu_{ij_i}$ would be sufficient for θ (Andersen, 1995).

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2. If we knew or had estimates of the ν_{ij_i} 's for all the items, then $\sum_i \nu_{ij_i}$ would be sufficient for θ (Andersen, 1995).

3. Suppose we know all the ν 's except for item i 's, then we could replace θ with a "rest-score",

$$\tilde{\theta}_{-i} = \phi \sum_{k \neq i} \nu_{kj_k}.$$

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- This gives us a multinomial logistic regression model where the predictor variable is a rest-score.

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$$\tilde{\theta}_{-i} = \phi \sum_{k \neq i} \nu_{kj_k}.$$

- This gives us a multinomial logistic regression model where the predictor variable is a rest-score.
- Justification and precedence comes from item response theory and classical test theory (Junker, 1993; Junker & Sijtsma, 2000).

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$$\text{Laws: } P(A = j_1 | \tilde{\theta}_A) = \exp[\lambda_{j_1}^A + \nu_{j_1}^A \phi(\nu_{j_2}^B + \nu_{j_3}^C + \nu_{j_4}^D)] \kappa_A$$

$$\text{Judges: } P(B = j_2 | \tilde{\theta}_B) = \exp[\lambda_{j_2}^B + \nu_{j_2}^B \phi(\nu_{j_1}^A + \nu_{j_3}^C + \nu_{j_4}^D)] \kappa_B$$

$$\text{House: } P(C = j_3 | \tilde{\theta}_C) = \exp[\lambda_{j_3}^C + \nu_{j_3}^C \phi(\nu_{j_1}^A + \nu_{j_2}^B + \nu_{j_4}^D)] \kappa_C$$

$$\text{Senate: } P(D = j_4 | \tilde{\theta}_D) = \exp[\lambda_{j_4}^D + \nu_{j_4}^D \phi^2(\nu_{j_1}^A + \nu_{j_2}^B + \nu_{j_3}^C)] \kappa_D$$

The κ 's ensure that probabilities over response options sum to 1 for each item.

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The κ 's ensure that probabilities over response options sum to 1 for each item.

Compatibility Conditions are needed to ensure this set is consistent with some joint distribution:

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$$\text{Senate: } P(D = j_4 | \tilde{\theta}_D) = \exp[\lambda_{j_4}^D + \nu_{j_4}^D \phi^2(\nu_{j_1}^A + \nu_{j_2}^B + \nu_{j_3}^C)] \kappa_D$$

The κ 's ensure that probabilities over response options sum to 1 for each item.

Compatibility Conditions are needed to ensure this set is consistent with some joint distribution:

When variable i is the response variable, the term relating variable i' to i must be the same as the term relating i to i' when i' is the response.

The proof is in paper for all models discussed in this talk.

The Joint Distribution

A set of multinomial logistic regression models that meet the compatibility conditions uniquely implies a log-multiplicative association model for the joint distribution.

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The Joint Distribution

A set of multinomial logistic regression models that meet the compatibility conditions uniquely implies a log-multiplicative association model for the joint distribution.

For the ANES data:

$$\begin{aligned}\log(P(\mathbf{y})) = & \lambda + \lambda_{j_1}^A + \lambda_{j_2}^B + \lambda_{j_3}^C + \lambda_{j_4}^D \\ & + \phi \left(\nu_{j_1}^A \nu_{j_2}^B + \nu_{j_1}^A \nu_{j_3}^C + \nu_{j_1}^A \nu_{j_4}^D + \nu_{j_2}^B \nu_{j_3}^C \right. \\ & \left. + \nu_{j_2}^B \nu_{j_4}^D + \nu_{j_3}^C \nu_{j_4}^D \right).\end{aligned}$$

A log-multiplicative association model.

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- Are special cases of Poisson regression models (i.e., log-linear models).

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Log-multiplicative Association Models

- Are special cases of Poisson regression models (i.e., log-linear models).
- Are generalizations of Goodman's (1979, 1986) multidimensional row-column or " $RC(M)$ " association models for 2-way tables.

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- Are implied by underlying multivariate normal distribution (Goodman, 1979; Becker, 1989; others).

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- Are special cases of Poisson regression models (i.e., log-linear models).
- Are generalizations of Goodman's (1979, 1986) multidimensional row-column or " $RC(M)$ " association models for 2-way tables.
- Are implied by underlying multivariate normal distribution (Goodman, 1979; Becker, 1989; others).
- Can be derived from a distance based model (de Rooij & Heiser, 2005), a generalization of Newton's Law of Gravity.

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- Are implied by underlying multivariate normal distribution (Goodman, 1979; Becker, 1989; others).
- Can be derived from a distance based model (de Rooij & Heiser, 2005), a generalization of Newton's Law of Gravity.
- Can be derived from a Formative latent variable model using statistical graphical models (Anderson & Böckenholt, 2000; Anderson & Vermunt, 2000; Anderson 2002; Anderson & Tettah, 2007).

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- Are generalizations of Goodman's (1979, 1986) multidimensional row-column or " $RC(M)$ " association models for 2-way tables.
- Are implied by underlying multivariate normal distribution (Goodman, 1979; Becker, 1989; others).
- Can be derived from a distance based model (de Rooij & Heiser, 2005), a generalization of Newton's Law of Gravity.
- Can be derived from a Formative latent variable model using statistical graphical models (Anderson & Böckenholt, 2000; Anderson & Vermunt, 2000; Anderson 2002; Anderson & Tettah, 2007).
- Can be derived from a Reflective latent variable model using standard item response theory methodology (Anderson & Yu, 2007; Anderson, Li, & Vermunt, 2007; Anderson, Verkuilen & Peyton, accepted).

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1. M Multiple (correlated) latent variables.

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● Extensions

- Multiple (correlated) latent variables
- Joint Distribution For M latent variables
- Restrictions on Parameters
- Hybrid Item Response Models
- Covariates

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To be discussed here. . .

1. M Multiple (correlated) latent variables.
2. Restrictions on parameters
 - Equality
 - Scaling
 - Other

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Extensions

To be discussed here. . .

1. M Multiple (correlated) latent variables.
2. Restrictions on parameters
 - Equality
 - Scaling
 - Other
3. Adding covariates
 - Information about response options.
 - Information about instructions (treatment condition).

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Multiple (correlated) latent variables

$$\tilde{\theta}_{m,-i} = \phi_{mm} \sum_{k \neq i} \nu_{kj_k m} + \sum_{m' \neq m} \phi_{m'm} \left(\sum_k \nu_{kj_k m'} \right)$$

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and the conditional multinomial model is

$$P(Y_i = j_i | \tilde{\theta}_{m,-i}, \forall m) = \frac{\exp[\lambda_{ij_i} + \sum_m \nu_{ij_i m} \tilde{\theta}_{m,-i}]}{\sum_{h_i} \exp[\lambda_{ih_i} + \sum_m \nu_{ih_i m} \tilde{\theta}_{m,-i}]}$$

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- If there is no direct relationship between an item and a latent variable, then the corresponding $\nu_{ij_i m} = 0$.

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- If there is no direct relationship between an item and a latent variable, then the corresponding $\nu_{ij_i m} = 0$.
- The compatibility conditions are

$$\phi_{mm'} \nu_{ij_i m} \nu_{kj_k m'} = \phi_{m'm} \nu_{kj_k m'} \nu_{ij_i m}$$

for all items, categories and latent variables.

i.e., $\phi_{mm'} = \phi_{m'm}$.

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Joint Distribution For M latent variables

The terms in the model include

1. A λ to ensure that probabilities summed over all possible response patterns equal 1.

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The terms in the model include

1. A λ to ensure that probabilities summed over all possible response patterns equal 1.
2. All marginal effects terms λ_{ij_i} (i.e., the intercepts in the multinomials).

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Applying these rules yields

$$P(\mathbf{y}) = \exp \left[\lambda + \sum_i \lambda_{ij_i} + \sum_i \sum_{k>i} \sum_m \sum_{m'} \phi_{mm'} \nu_{ij_i m} \nu_{kj_k m'} \right]$$

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Restrictions on Parameters

- Equality and ordinal restrictions on category scores are straight forward.

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Restrictions on Parameters

- Equality and ordinal restrictions on category scores are straight forward.

- For identification, need to set the

- ◆ Location the λ_{ij_i} and category scores ν_{ij_i} , e.g.,

$$\sum_{j_i} \lambda_{ij_i} = \sum_{j_i} \nu_{ij_i} = 0$$

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- ◆ Scale of one set of ν_{ij_i} per latent variable, e.g.,

$$\sum_{j_i} \nu_{ij_i}^2 = 1$$

- ◆ The category scale can be re-expressed as

$$\nu_{ij_i} = \omega_i \nu_{ij_i}^*$$

where $\omega_i = \sqrt{\sum_{j_i} \nu_{ij_i}^2}$.

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Hybrid Item Response Models

- The category scale can be re-expressed as

$$v_{ij_i} = \omega_i v_{ij_i}^*$$

where $\omega_i = \sqrt{\sum_{j_i} v_{ij_i}^2}$.

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Hybrid Item Response Models

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where $\omega_i = \sqrt{\sum_{j_i} v_{ij_i}^2}$.

- Set some (or all) $\omega_i = 1$ yields a “hybrid” item response model where
 - ◆ Each item is equally strongly related to the latent variable (Rasch-like).

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 - ◆ Each item is equally strongly related to the latent variable (Rasch-like).
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Hybrid Item Response Models

- The category scale can be re-expressed as

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where $\omega_i = \sqrt{\sum_{j_i} \nu_{ij_i}^2}$.

- Set some (or all) $\omega_i = 1$ yields a “hybrid” item response model where
 - ◆ Each item is equally strongly related to the latent variable (Rasch-like).
 - ◆ The category scores $\nu_{ij_i}^*$ carry category specific information (Bock NRM-like).
- The restriction of equal $\omega_i (= 1)$ for items can be imposed by placing scaling restriction on the category scores; that is,

$$\sum_{j_i} \nu_{ij_i}^2 = 1$$

on categories scores for more items than what is needed for identification.

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Covariates

Covariates can be added to the model by creating or modifying sub-models for the intercepts and/or latent variable(s).

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Covariates

Covariates can be added to the model by creating or modifying sub-models for the intercepts and/or latent variable(s).

■ Possible models for the intercept:

$$\lambda_{ij_i} = \lambda_{ij_i}^* + \sum_q \beta_{ij_i q} x_q$$

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- A possible model for a latent variable (i.e., covariate adds information about a person's value on the latent variable):

$$\tilde{\theta}_{-i} = \phi \left(\sum_{k \neq i} \nu_{kj_k} + \sum_q \eta_q x_q \right).$$

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Analysis of the ANES Data

1. Hypotheses regarding “Don’t know”:

- Mondak (2001): “Don’t know” responders might have intermediate states of knowledge but be unwilling to venture a substantive response and thus be scored wrong even though they knew more than responders who answer incorrectly.

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1. Hypotheses regarding “Don’t know”:

- Mondak (2001): “Don’t know” responders might have intermediate states of knowledge but be unwilling to venture a substantive response and thus be scored wrong even though they knew more than responders who answer incorrectly.
- “Don’t know” responders have less knowledge than those who respond with a substantive but incorrect answer.

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- “Don’t know” responses are simply not comparable to substantive responses, be they correct or incorrect.

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- “Don’t know” responses are simply not comparable to substantive responses, be they correct or incorrect.

2. Instructions given to respondents would effect the way they responded. Treatment condition:

- Standard or control group ($n = 606$)
- Instructions that encouraged respondents to guess if they were not sure of the correct answer ($n = 570$).

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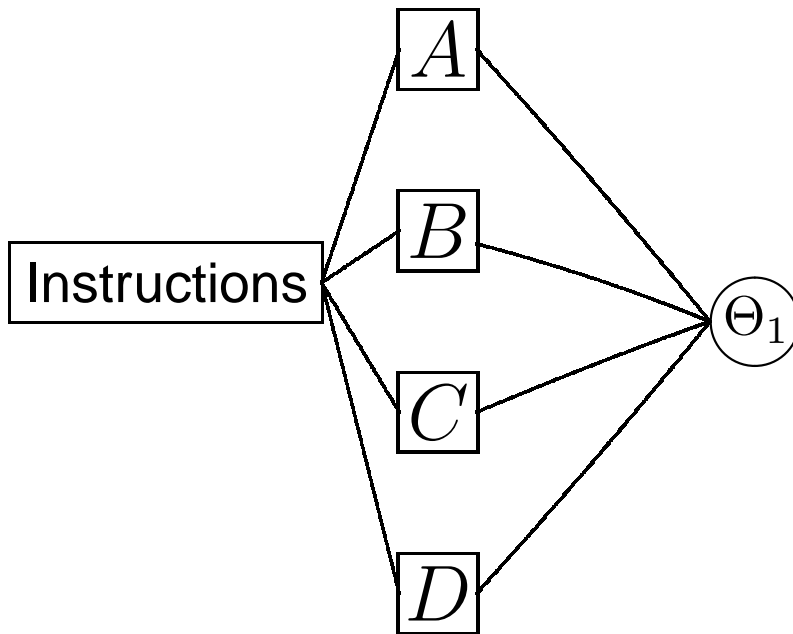
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3. Goal: Measure political knowledge.

Possible Graphs for the ANES Data



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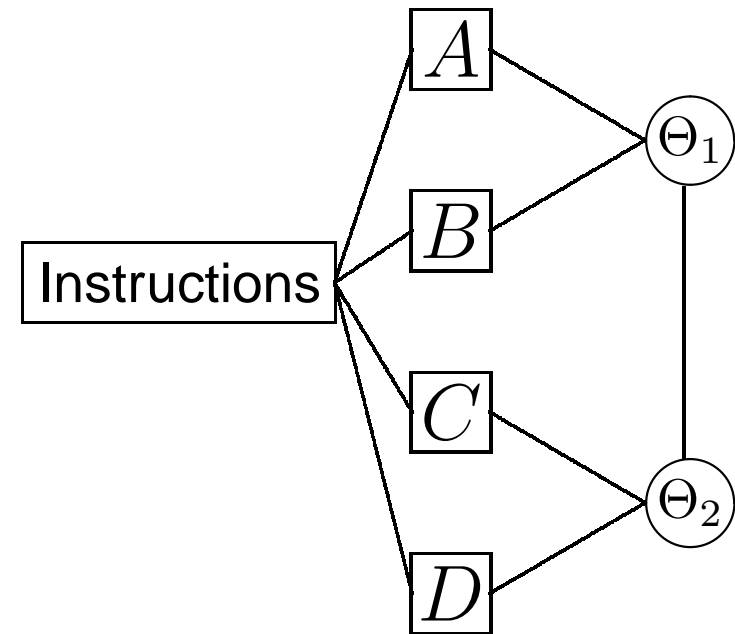
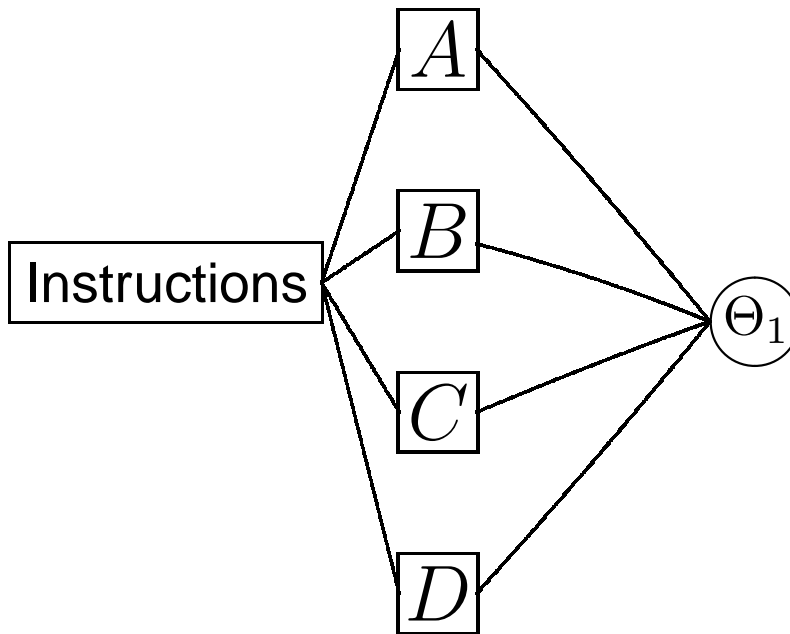
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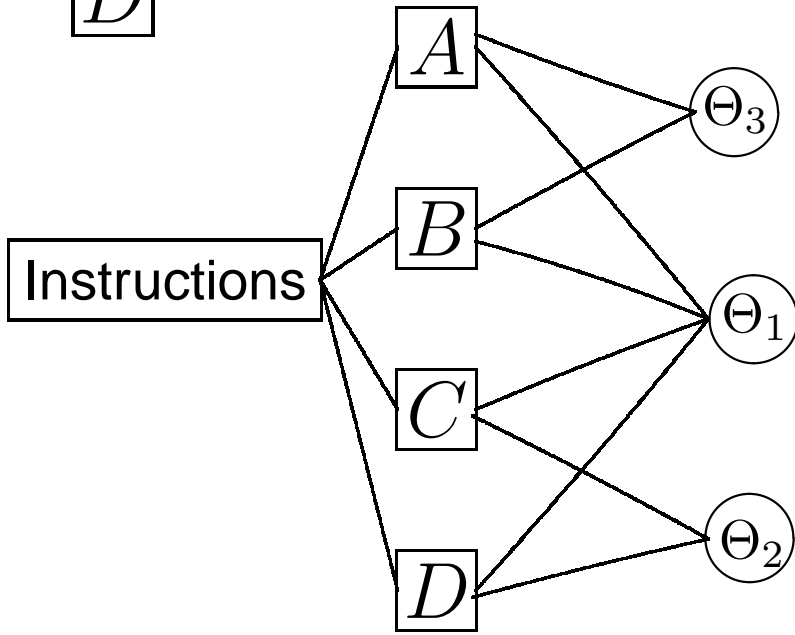
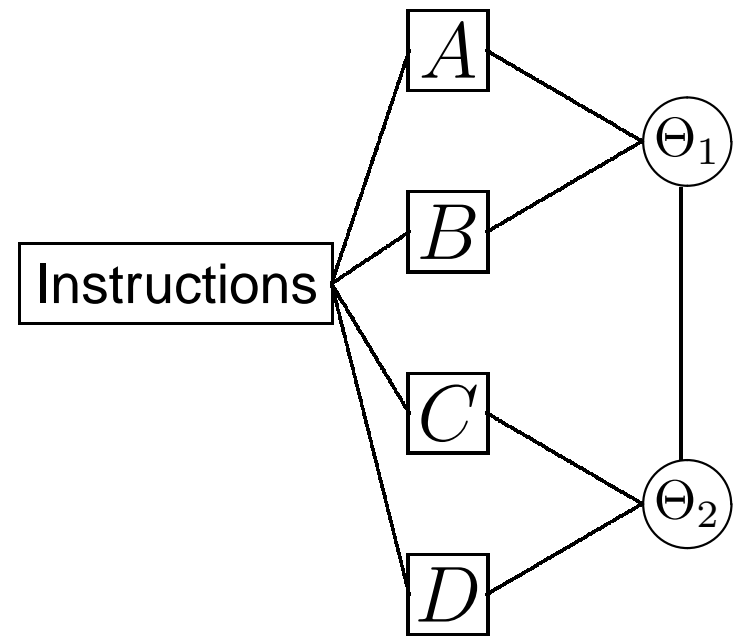
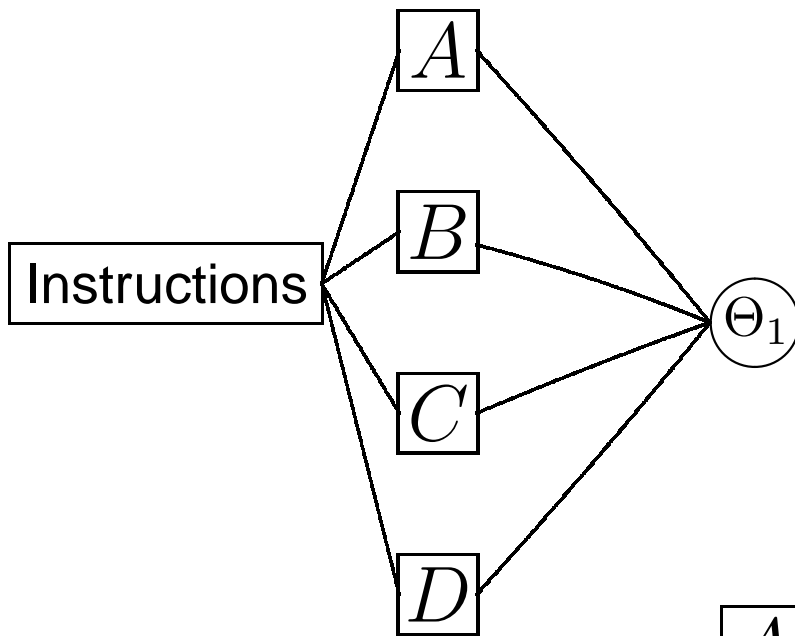
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Analysis of the ANES Data

Fit statistics for models fit to the cross-classification of instructions and responses to the four ANES items.

Model	# par	G^2	BIC	Percent account
Log-linear Models				
Independence	11	1533.02	-418.27	—
All 2-way	58	189.66	-1429.34	88%
Log-multiplicative association models (LMA)				
One dimension	31	418.22	-1391.67	73%
Two dimensions	32	312.88	-1489.94	80%
2D Hybrid	30	313.12	-1503.84	80%
2D Hybrid with $\nu_{11}^A = \nu_{21}^A$	29	316.79	-1507.23	79%
Three dimensional LMA	38	227.54	-1532.86	85%
3D Hybrid	36	229.69	-1544.84	85%
3D Hybrid w/ $\nu_{11}^A = \nu_{21}^A$	35	230.33	-1551.28	85%
3D Hybrid w/ $\nu_{11}^A = \nu_{21}^A, \nu_{j32}^C = \nu_{j32}^D$	34	230.34	-1558.33	85%

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Scoring and “Don’t Know”

The estimated scale values/scores:

Item	Category	Parameter	Estimate
A: Determines constitutionality of a law	President	ν_{11}^A	-0.036
	Congress	ν_{21}^A	-0.036
	Supreme Ct.	ν_{31}^A	0.741
	Don't know	ν_{41}^A	-0.700
B: Responsible for nominating federal judges	President	ν_{11}^B	0.775
	Congress	ν_{21}^B	0.095
	Supreme Ct.	ν_{31}^B	-0.359
	Don't know	ν_{41}^B	-0.511
C: Holds majority in the House	Republican	ν_{12}^C	0.549
	Democrat	ν_{322}^C	0.249
	Don't know	ν_{32}^C	-0.798
D: Holds majority in the Senate	Republican	ν_{12}^D	0.625
	Democrat	ν_{22}^D	0.143
	Don't know	ν_{32}^D	-0.768

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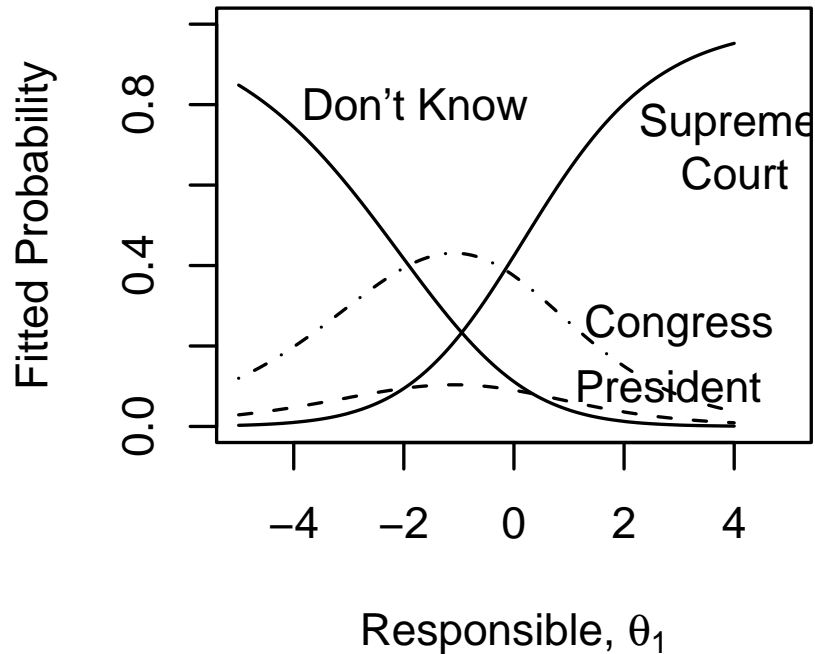
● Scoring and "Don't Know"

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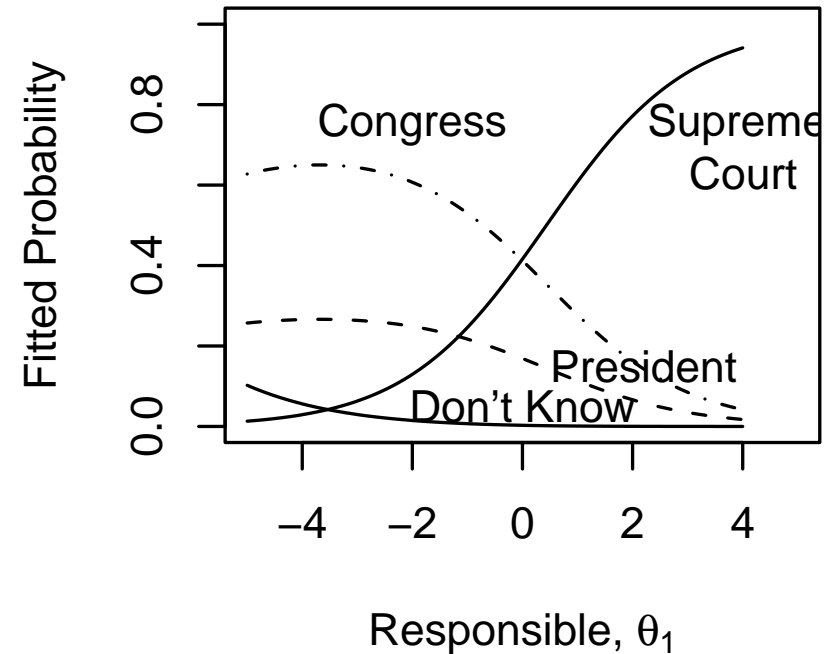
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Standard Instructions



Encourage Guessing



Single Measure of Political Knowledge

and another look at the effect of Instructions

Scoring Method	Cronbach's alpha (95% CI's)			
	Control Group		Guessing Group	
Binary	.68	(.64, .72)	.58	(.53, .63)
LMA category scores	.75	(.68, .82)	.68	(.60, .75)

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Cronbach's alpha (95% CI's)

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LMA category scores	.75	(.68, .82)	.68	(.60, .75)

Comments:

- Scores from all the LMA models are all very similar.

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- Scores from all the LMA models are all very similar.
- Model based scores lead to larger reliabilities.

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	Control Group		Guessing Group	
Binary	.68	(.64, .72)	.58	(.53, .63)
LMA category scores	.75	(.68, .82)	.68	(.60, .75)

Comments:

- Scores from all the LMA models are all very similar.
- Model based scores lead to larger reliabilities.
- Less reliable measures when guessing is encouraged.

ANES Data

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Approach

Extensions

Analysis of the ANES Data

- Analysis of the ANES Data
- Possible Graphs for the ANES Data
- Analysis of the ANES Data
- Scoring and "Don't Know"
- Effect of Instructions: Constitutionality of Laws
- **Single Measure of Political Knowledge**

Conclusion & Beyond

Conclusion & Beyond

- LMA models are effective tools for analysis and measurement of polytomous response data where latent variables may underling responses.

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● Higher-Level Factors

- LMA models are effective tools for analysis and measurement of polytomous response data where latent variables may underling responses.
- **Estimation** that's more flexible and capable of larger problems.
 - ◆ ℓ_{EM} can handle about 2^{12} .
 - ◆ SAS PROC NLP can handle about 2^{20} .
 - ◆ Pseudolikelihood of models in the Rasch family can handle at least 5^{100} .
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Conclusion & Beyond

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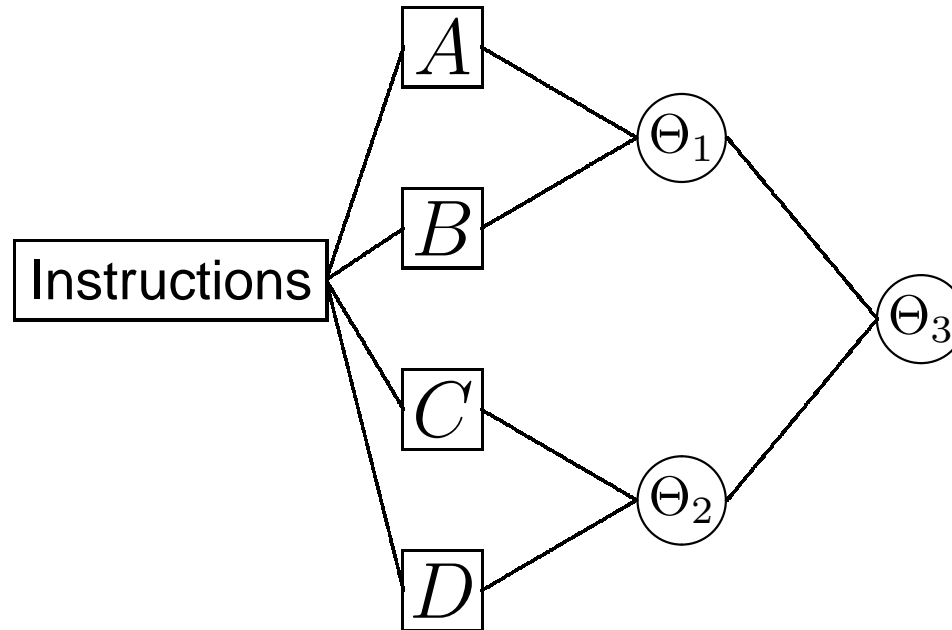
● Conclusion & Beyond

● Higher-Level Factors

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- **Some Additional Possibilities:**
 - ◆ Other transformations of scale values, including ordinal and partial ordinal restrictions, spline, linear.
 - ◆ Higher level factors (i.e., lower rank decomposition of matrix of ϕ 's).

Higher-Level Factors

This model did not fit the ANES data



i.e., $\phi_{11} = l_1^2$, $\phi_{12} = l_1 l_2$, and $\phi_{22} = l_2^2$.

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